Testing our understanding of polynomials and splines in Bézier form

1 affine invariance

[2] Is the power form of a polynomial affine invariant? (If yes, give a proof, if no, give a counterexample.)

2 2D Curves

[4] A planar polynomial curve piece p in Bézier form on the interval [0..1] has the coefficients

$$\begin{bmatrix} -2 \\ 8 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 8 \end{bmatrix}.$$

What is the degree of this polynomial piece? Compute the position $p(t)$ at $t = 1/2$ by DeCasteljau's algorithm. What is the normal of p (in the plane) at $t = 1/2$?

(b) [4 points] The curve ℓ has Bézier coefficients $(\ell_0, \ldots, \ell_m)$ and the curve **r** has Bézier coefficients $(r_0, \ldots, r_m)$. With

$$\ell_{m-2} := \begin{bmatrix} -2 \\ -1 \end{bmatrix},\ \ell_{m-1} := \begin{bmatrix} -1 \\ 0 \end{bmatrix},\ r_1 := \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

determine $\ell_m = r_0$ and r_2 so that the concatenated curve ℓ, r is twice continuously differentiable.

3 2D Curves

[4] A 'left' polynomial curve piece a in Bézier form on the interval [0..1] has the coefficients

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ y_3 \end{bmatrix}$$

and a 'right' polynomial curve piece c in Bézier form on the interval [0..1] has the coefficients

$$\begin{bmatrix} 6 \\ y_6 \end{bmatrix} \begin{bmatrix} 7 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 1 \end{bmatrix}.$$

Determine the ordinates y_3, y_4, y_5 and y_6 of a polynomial curve piece b in Bézier form on the interval [0..1] and with coefficients

$$\begin{bmatrix} 3 \\ y_3 \end{bmatrix} \begin{bmatrix} 4 \\ y_4 \end{bmatrix}, \begin{bmatrix} 5 \\ y_5 \end{bmatrix}, \begin{bmatrix} 6 \\ y_6 \end{bmatrix}$$

so that a, b and c join with maximal derivative continuity. (Hint, draw a sketch, set up the equations)

4 Area computation

Consider a unit square. Place the control points of each of four consecutive Bézier segments of degree 2 at the middle of an edge, the following vertex and the following mid-edge. What is the area enclose by the four Bézier segments?

5 Multivariate Bézier form

[2+2] The domain simplex (equilateral triangle) of a bi-variate polynomial in Bézier form can be chopped into four equal domain pieces. How about the domain simplex (tetrahedron) of a tri-variate polynomial in Bézier form?

In a bi-variate polynomial in Bézier form of degree 3, one coefficient does not influence the polynomial piece on the boundaries of the domain. What is it subscript? In a tri-variate polynomial in Bézier form of degree 3, does there exist a coefficient that does not influence the polynomial piece on the boundaries of the domain?